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**Exercises**

Section 3.1: 3, 8, 14, 15, 16, 19 (b,c,d)

3. Demonstrate graphically that the equation 50π + sin x = 100 arctan x has infinitely many solutions.

Answer:

8. If a = 0.1 and b = 1.0, how many steps of the bisection method are needed to determine the root with an error of at most 1 /2 × 10−8?

Answer:

14. Denote the successive intervals that arise in the bisection method by [a0, b0], [a1, b1], [a2, b2], and so on.

a. Show that a0 ≤ a1 ≤ a2 ··· and that b0 ≥ b1 ≥ b2 ···.

b. Show that bn − an = 2−n(b0 − a0).

c. Show that, for all n, anbn + an−1bn−1 = an−1bn + anbn−1.

Answer:

15. (Continuation) Can it happen that a0 = a1 = a2 =···

Answer:

16. (Continuation) Let cn = (an + bn)/2. Show that

lim n→∞ cn = lim n→∞ an = lim n→∞ bn .

Answer:

19. (True–False) Using the notation of the text, determine which of these assertions are true and which are generally false:

b. an ≤ r ≤ cn

c. cn ≤ r ≤ bn

d. |r − an| ≤ 2−n

Answer:

Section 3.2: 5, 6, 14

5. The equation x − Rx−1 = 0 has x = ±R1/2 for its solution. Establish Newton’s iterative scheme, in simplified form, for this situation. Carry out five steps for R = 25 and x0 = 1.

Answer:

6. Using a calculator, observe the sluggishness with which Newton’s method converges in the case of f (x) = (x − 1)m with m = 8 or 12. Reconcile this with the theory. Use x0 = 1.1.

Answer:

14. Determine the formulas for Newton’s method for finding a root of the function f (x) = x − e/x. What is the behavior of the iterates?

Answer:

Section 3.3: 2, 5, 13 (b,d)

2. If we use the secant method on f (x) = x 3 − 2x + 2 starting with x0 = 0 and x1 = 1, what is x2?

Answer:

5. Using the bisection method, Newton’s method, and the secant method, find the largest positive root correct to three decimal places of x 3 − 5x + 3 = 0. (All roots are in [−3, +3].)

Answer:

13. Test the following sequences for different types of convergence (i.e., linear, superlinear, or quadratic), where n = 1, 2, 3 ... .

b. xn = 2−n

d. xn = 2−an with a0 = a1 = 1 and an+1 = an + an−1 for n ≥ 2.

Answer:

**Computing Exercises**

Section 3.1: 3, 7

3. Find a root of the equation tan x = x on the interval [4, 5] by using the bisection method. What happens on the interval [1, 2]?

Answer:

7. Use the bisection method to determine roots of these functions on the intervals indicated. Process all three functions in one computer run.

f (x) = x 3 + 3x − 1 on [0, 1]

g(x) = x 3 − 2 sin x on [0.5, 2]

h(x) = x + 10 − x cosh(50/x) on [120, 130]

Answer:

Section 3.2: 2, 6, 15, 18

2. Write a simple, self-contained program to apply Newton’s method to the equation x 3 + 2x 2 + 10x = 20, starting with x0 = 2. Evaluate the appropriate f (x) and f’ (x), using nested multiplication. Stop the computation when two successive points differ by 1/ 2 × 10−5 or some other convenient tolerance close to your machine’s capability. Print all intermediate points and function values. Put an upper limit of ten on the number of steps.

Answer:

6. In celestial mechanics, **Kepler’s equation** is important. It reads x = y − ε sin y, in which x is a planet’s mean anomaly, y its eccentric anomaly, and ε the eccentricity of its orbit. Taking ε = 0.9, construct a table of y for 30 equally spaced values of x in the interval 0 ≤ x ≤ π. Use Newton’s method to obtain each value of y. The y corresponding to an x can be used as the starting point for the iteration when x is changed slightly.

Answer:

15. Find the root of the equation 1 /2 x 2 + x + 1 − ex = 0 by Newton’s method, starting with x0 = 1, and account for the slow convergence.

Answer:

18. Using the bisection method, find the positive root of 2x(1 + x 2)−1 = arctan x. Using the root as x0, apply Newton’s method to the function arctan x. Interpret the results.

Answer:

Section 3.3: 7, 10

7. Write a simple program to find the root of f (x) = x 3 + 2x 2 + 10x − 20 using the secant method with starting values x0 = 2 and x1 = 1. Let it run at most 20 steps, and include a stopping test as well. Compare the number of steps needed here to the number needed in Newton’s method. Is the convergence quadratic?

Answer:

10. Test the secant method on an example in which r, f’ (r), and f’’ (r) are known in advance. Monitor the ratios en+1/(enen−1)to see whether they converge to −1/ 2 f (r)/ f’ (r).

Answer: